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GAS DYNAMIC CONTROL OF SPACE VEHICLE MOVEMENT BY BANK IN THE AT--ETC(U)
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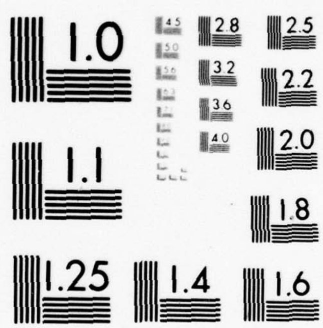

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FOREIGN TECHNOLOGY DIVISION

GAS DYNAMIC CONTROL OF SPACE VEHICLE MOVEMENT BY BANK IN THE ATMOSPHERE

by

R. V. Studnev

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
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М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yë or ë.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

1814

GAS DYNAMIC CONTROL OF SPACE VEHICLE MOVEMENT BY BANK IN THE ATMOSPHERE

R. V. Studnev (USSR)

At present one of the urgent problems is that of control of a space vehicle, possessing aerodynamic quality, during reentry. In a whole series of works there is being examined atmospheric entry with vehicle balanced at constant angle of attack, trajectory control of which is accomplished by change of the angle of bank [1, 2]. In connection with this appears the problem of evaluation of the dynamic possibilities of movement of a space vehicle relative to the center of mass with compensation of disturbances in terms of angles of attack and slip (α, β), and also with control of bank angle. There is known a whole series of works, dedicated to the analysis of optimum control of orientation of the space vehicle during movement in a void [3-6]. The majority of these problems was solved with the use of the principle of Pontryagin maximum [7, 8]. Below, on the basis of the

maximum principle, in simplified form is solved a similar problem of optimum control of orientation of a space vehicle in the atmosphere, which is somewhat complicated in comparison with the problems of motion in a void because of the necessity of considering the aerodynamic moments of stability of the vehicle.

1. Control of motion of space vehicle relative to center of mass during atmospheric entry.

We will examine movement of a space vehicle relative to the center of mass in approximate formulation, disregarding the interference with movement of the center of mass. We will also consider that during the execution of bank turns and compensation of deviations with respect to α and β the parameters of motion of the vehicle (V, q) are not substantially changed and equations of motion can be considered as equations with "frozen" coefficients. Finally, we will consider the motion of the space vehicle relative to the center of mass sufficiently slow, so that in equations of motion the nonlinear terms of type $\omega_y\omega_x$, $\beta\omega_x$ etc., could be disregarded. TP With the noted assumptions the equations, recorded in the principal central axes of inertia ($OX_1Y_1Z_1$) (Fig. 1), will have the form:

$$\dot{\omega}_z = \overline{M}_z^{\alpha}\alpha + u_z, \qquad \dot{\alpha} + \omega_z, \tag{1.1}$$

$$\begin{aligned} \dot{\omega}_{y_1} &= \overline{M}_y^{\beta}\beta + u_y, & \dot{\beta} &= \cos\alpha_0 \cdot \omega_y + \sin\alpha_0 \cdot \omega_x, \\ \dot{\omega}_{x_1} &= \overline{M}_x^{\beta}\beta + u_x, & \dot{\gamma} &= \cos\alpha_0 \cdot \omega_x - \sin\alpha_0 \cdot \omega_y, \end{aligned} \tag{1.2}$$

where α_0 - balanced angle of attack of space vehicle (α_0=const); u_x, u_y, u_z - moments from the controls, pertaining to corresponding moments of inertia.

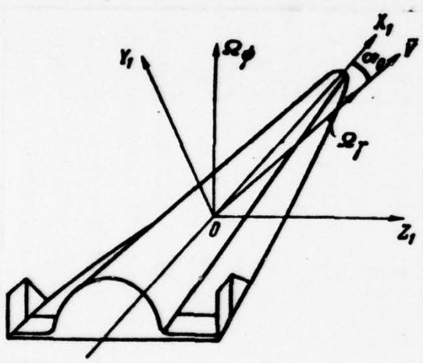


Fig. 1.

In equations (1.1) and (1.2) are preserved only the moments of aerodynamic stability of the vehicle, since at hypersonic speeds the effect of aerodynamic damping can be disregarded. From (1.1) and (1.2) it follows that the equations of three-dimensional motion of the space vehicle are divided into equations of longitudinal (1.1) and lateral (1.2) motions and they can be investigated separately.

Let us examine the equations of lateral motion of a space

vehicle (1.2). Let us convert these equations so as to separate the motion of the vehicle in terms of bank (rotation around velocity vector $\overline{V}$) and yaw (angle β). Let us multiply the first and second equations (1.2) respectively by $\cos\alpha_0$ and $\sin\alpha_0$ and having summarized, we obtain the equation for $\dot{\Omega}_\psi$, and after their multiplication respectively by $(-\sin\alpha_0)$ and $\cos\alpha_0$ and summation - the equation for

$$\begin{aligned} &\dot{\Omega}_\psi = \sigma_\beta\beta + u_\psi, \qquad \dot{\beta} = \Omega_\psi, \\ &\dot{\Omega}_\gamma = \sigma_\gamma\beta + u_\gamma, \qquad \dot{\gamma} = \Omega_\gamma. \end{aligned} \tag{1.3}$$

In equations (1.3) are accepted the following designations:

$$\begin{aligned} &\Omega_\psi = \cos\alpha_0\cdot\omega_y + \sin\alpha_0\cdot\omega_x, \\ &\Omega_\gamma = \cos\alpha_0\cdot\omega_x - \sin\alpha_0\cdot\omega_y, \end{aligned} \tag{1.4}$$

$$\begin{aligned} &\sigma_\beta = \overline{M}_y^\beta\cos\alpha_0 + \overline{M}_x^\beta\sin\alpha_0, \\ &\sigma_\gamma = \overline{M}_x^\beta\cos\alpha_0 - \overline{M}_y^\beta\sin\alpha_0; \end{aligned} \tag{1.5}$$

$$\left.\begin{aligned} &u_\gamma = u_x\cos\alpha_0 - u_y\sin\alpha_0; \\ &u_\psi = u_y\cos\alpha_0 + u_x\sin\alpha_0. \end{aligned}\right\} \tag{1.6}$$

From equations (1.3) it follows that motion of the space vehicle with respect to angle β does not depend on angle γ and control u_γ.

Let us examine the selection of controls u_x, u_y, at which u_γ and u_ψ are independent. This can be achieved either by coordinated deflection of aerodynamic controls (u_x, u_y), or by special orientation of control jet engines with gas-dynamic control of the vehicle.

Let us explain how it is necessary to set the control jet engines on the vehicle, so that one pair would create only moment

u_{γ}, and the other - only moment u_{ψ}.

The control jet engine with thrust P_1, the vector of which is arranged in a plane, parallel with plane OY_1Z_1, installed on the stern of the vehicle at angle ϕ_1 to axis OZ_1, creates adjusted moments relative to axes OX_1 and OY_1, determined by formulas (Fig. 2)

$$U_{x_1} = 2\frac{P_1 \sin\varphi_1 l_x}{I_x}, \qquad u_{y_1} = 2\frac{P_1 \cos\varphi_1 l_y}{I_y}, \qquad (1.7)$$

where l_x, l_y - distance of one controlled jet engine to axes OX_1 and OY_1 respectively; I_x, I_y - principal moments of inertia of the space vehicle relative to axes OX_1 and OY_1.

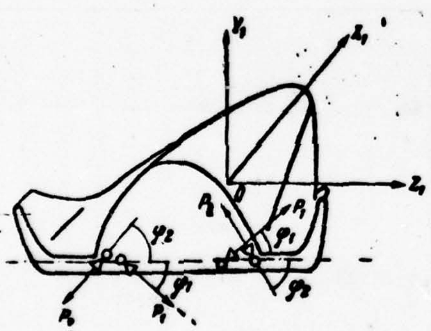


Fig. 2.

Let us select the setting angle of the control jet engine ϕ_1 from the condition that $u_{\gamma} = 0$ at $P_1 \neq 0$. By placing expressions (1.7) in relations (1.6) and equating the first relation of (1.6) to zero, we obtain the condition for ϕ_1 in the form

$$\varphi_1 = \mathrm{arctg}\left(\frac{l_y I_x}{l_x I_y}\,\mathrm{tg}\,\alpha_0\right). \tag{1.8}$$

With such selection of ϕ_1 equation u_ψ is determined by formula

$$u_\psi = \frac{2P_1 l_y}{I_y}\,\frac{1}{\cos\alpha_0\sqrt{1+\left(\frac{l_y I_x}{l_x I_y}\,\mathrm{tg}\,\alpha_0\right)^2}}. \tag{1.9}$$

In a particular case, when $l_y I_x / l_x I_y = 1$, the expression for u_ψ is simplified.

$$u_\psi = \frac{2P_1 l_y}{I_y}. \tag{1.10}$$

Analogously from condition $u_\gamma = 0$ is found orientation of the control jet engines of the second pair (Fig. 2):

$$\varphi_2 = -\,\mathrm{arctg}\left(\frac{l_y I_x}{l_x I_y}\,\frac{1}{\mathrm{tg}\,\alpha_0}\right). \tag{1.11}$$

From (1.8) and (1.11) it follows that generally the control jet engines are oriented not orthogonally to each other, but make up angle $\phi_1 - \phi_2$, the tangent of which is found by formula

$$\mathrm{tg}\,(\varphi_1 - \varphi_2) = \frac{2}{\sin 2\alpha_0}\left[\frac{l_y I_x}{l_x I_y}\,\frac{1}{1-\left(\frac{l_y I_x}{l_x I_y}\right)^2}\right]. \tag{1.12}$$

With $l_y I_x / l_x I_y = 1$ the control jet engines, creating moments u_γ and u_ψ, are orthogonal $[(\phi_1 - \phi_2) = 90°]$. In Fig. 3 as an example is constructed the dependence of $(\phi_1 - \phi_2)$ on $l_y I_x / l_x I_y$ for $\alpha_0 = 30°$.

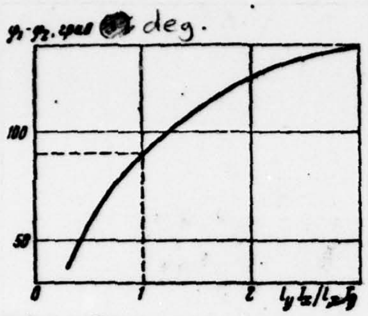


Fig. 3.

Analogous conversions can be performed during the analysis of aerodynamic control. It is easy to show that for control of yaw (u_ψ) moments u_x and u_y should be connected by relationship

$$u_x / u_y = \operatorname{tg} \alpha_0, \tag{1.13}$$

in this case

$$|u_\psi| = 2 |u_y| \frac{1}{\cos \alpha_0}. \tag{1.14}$$

For control of bank we obtain that u_x and u_y should be connected by relationship

$$u_x / u_y = -1 / \operatorname{tg} \alpha_0, \tag{1.15}$$

in this case

$$|u_\gamma| = 2 |u_x| \frac{1}{\cos \alpha_0}. \tag{1.16}$$

For convenience of analysis and the obtaining of more general results let us convert equations to dimensionless form. Let us

introduce dimensionless time τ with the aid of relationship

$$d\tau = \sqrt{-\sigma\beta}\,dt. \tag{1.17}$$

Let us change the scales of independent variables, considering limitations on control u_ϕ and u_γ

$$|u_\phi| \leqslant |U_\phi|, \quad |u_\gamma| \leqslant |U_\gamma|, \tag{1.18}$$

for which let us introduce the following designations:

$$\begin{aligned} &\bar{\beta} = \beta \frac{-\sigma\beta}{|U_\phi|}, && \bar{\Omega}_\phi = \frac{\Omega_\phi \sqrt{\sigma\beta}}{|U_\phi|}, \\ &\sigma_\gamma^* = \frac{\sigma_\gamma |u_\phi|}{(-\sigma_\beta)|U_\gamma|}, && \Omega_\gamma = \frac{\Omega_\gamma \sqrt{-\sigma\beta}}{|U_\gamma|}, \\ &\bar{\gamma} = \frac{\gamma(-\sigma_\beta)}{|U_\gamma|}. \end{aligned} \tag{1.19}$$

Taking into account designations (1.19) the equations of motion take the form:

$$\begin{aligned} &\bar{\Omega}_\phi' = -\bar{\beta} + \bar{u}_\phi, \\ &\bar{\beta}' = \bar{\Omega}_\phi, \ |\bar{u}_\phi| \leqslant 1; \end{aligned} \tag{1.20}$$

$$\begin{aligned} &\bar{\Omega}_\gamma' = \sigma_\gamma^* \bar{\beta} + \bar{u}_\gamma, \\ &\bar{\gamma}' = \bar{\Omega}_\gamma, |\bar{u}_\gamma| \leqslant 1. \end{aligned} \tag{1.21}$$

Let us turn now to analysis of the optimum control of the space vehicle with the use of equations (1.20) and (1.21).

2. Investigation of the form of p-trajectories.

In accordance with the principal of Pontryagin maximum [7, 8] let us compile function

$$H = -p_1\bar{\beta} + p_1\bar{u}_\psi - p_2\bar{\Omega}_\psi + p_3\overline{\sigma_\gamma^*}\beta + p_3\bar{u}_\gamma + p_4\bar{\Omega}_\gamma. \tag{2.1}$$

Let us write out the terms of functions H, containing control:

$$H_u = p_1\bar{u}_\psi + p_3\bar{u}_\gamma. \tag{2.2}$$

From the condition of maximum of H with respect to u_j it follows that controls $\bar{u}_\psi$ and $\bar{u}_\gamma$ are Rayleigh functions, determined from condition

$$\bar{u}_\psi = \operatorname{sign} p_1,\ \bar{u}_\gamma = \operatorname{sign} p_3, \qquad (|\bar{u}_\psi| = |\bar{u}_\gamma| = 1). \tag{2.3}$$

For finding the optimum control it is necessary to find the solution of system of conjugate equations $p_i' = -\partial H/\partial X_i$.

Let us compile a system of conjugate equations and examine the possible forms of solution of this system. Conjugate equations have the following form:

$$p_1' = -p_2, \qquad p_2' = p_1 - p_3\sigma_\gamma^*; \tag{2.4}$$
$$p_3' = -p_4, \qquad p_4' = 0. \tag{2.5}$$

Equations for p_3' and p_4' are easily integrated:

$$p_3 = -c_4\tau + c_3, \qquad p_4 = c_4. \tag{2.6}$$

From solution of (2.6) for $p_3(\tau)$ and condition (2.3) it follows that generally control $\bar{u}_\gamma$ can change sign not more than once.

Conjugate variables $p_1, \ldots, p_4$ are determined taking into account boundary conditions, imposed on the actual variables. In view of the linearity of conjugate equations and the Rayleigh character of control functions the conjugate variables are determined with accuracy to an arbitrary constant multiplier. In connection with this from (2.4) it follows that the type of solution for p_1 and p_2 does not depend on coefficient σ_γ^0, since change of this coefficient is equivalent to change of the scale of variable p_3. Only the solution for actual variables depends on the coefficient σ_γ^0.

Let us examine in more detail the solution for conjugate variables p_1, p_2. Considering the form of the solution for $p_3(\tau)$, with the aid of conversion of variables

$$\bar{p}_1 = p_1 - c_3\sigma_\gamma^0 + c_4\sigma_\gamma^0\tau, \qquad \bar{p}_2 = p_2 - c_4\sigma_\gamma^0, \tag{2.7}$$

we obtain the equations for $\bar{p}_1$ and $\bar{p}_2$, which are easily integrated:

$$\bar{p}_1' = -\bar{p}_2, \qquad \bar{p}_2' = \bar{p}_1. \tag{2.8}$$

The solution of this system of equations on phase plane p_1p_2 is a family of concentric circumferences (Fig. 4, a).

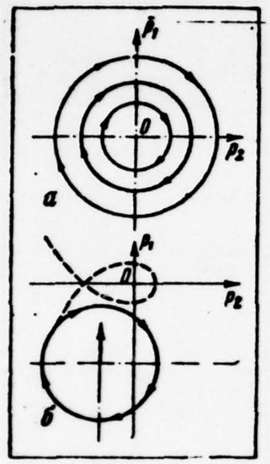


Fig. 4.

For analysis of optimum control it is necessary to examine the change in time only of function $p_1(\tau)$ (values of function p_2 are unessential). From relations (2.7) and (2.8) it follows that change of variables p_1 and p_2 on the phase plane can be represented as the result of two movements: movement of symbolic point along the circumference and displacement of the circumference (Fig. 4, b). The possible types of such curves are illustrated by Fig. 5.

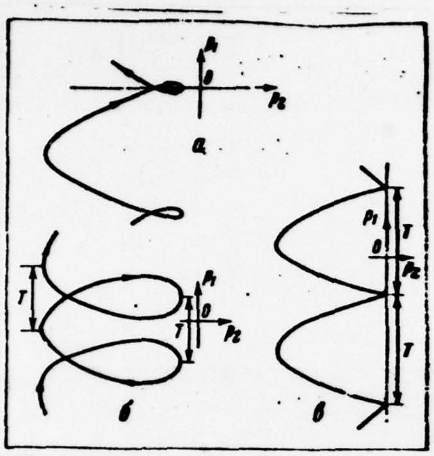


Fig. 5.

Let us note some properties of the obtained solutions for conjugate variables (p_1, p_2). It is possible to show that phase curves are symmetric relative to axis p_1=const, passing through the points, in which $\partial p_1/\partial p_2 = 0$.

In order to prove this, let us examine the projections of the velocity of symbolic point to vertical (Op_1) and horizontal (Op_2) axes. These projections are equal to (Fig. 6):

$$V_{p_1} = -R\cos\varphi + \frac{2a}{T}, \qquad V_{p_2} = R\sin\varphi, \tag{2.9}$$

where R - radius of circumference of p-trajectory; 2a/T - rate of displacement of the center of circumference.

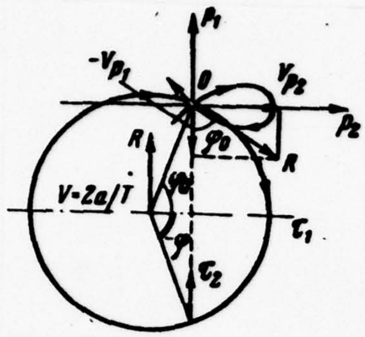


Fig. 6.

Using the expressions for projections of velocity V_{p_1} and V_{p_2}, we obtain derivative $\partial p_1/\partial p_2$:

$$\frac{\partial p_1}{\partial p_2}=\frac{V_{p_1}}{V_{p_2}}=\frac{-R\cos\varphi+\frac{2a}{T}}{R\sin\varphi}. \qquad (2.10)$$

From (2.10) it follows that derivatives $\partial p_1/\partial p_2$ with values of ϕ equal in value, but different in sign, have different values and are different in sign. In particular, derivative $\partial p_1/\partial p_2$ is changed into zero with value of ϕ, determined from relation

$$\varphi=\pm\arc\cos\frac{2a}{TR}. \qquad (2.11)$$

and approaches infinity with ϕ = 0 and ϕ = 180°. Such character of change of derivatives indicates the fact that the phase curve has axis of symmetry, corresponding to angle ϕ = 0.

The time of motion of symbolic point between the points of

contact of phase curve with generatrix p_2 = const is equal to the period of revolution of the point along the circumference (i.e. 2π), and the distance between the points of contact is identical for both generatrices (Fig. 5, b).

In the region, where the direction of motion along the circumference and displacement of the center of circumference are opposite, the phase curve $p_1(p_2)$ can have a loop. In the case where the speed of displacement of the center of the circumference is greater than the speed of motion of the symbolic point along the circumference, the loop on the phase trajectory disappears.

Let us estimate the time of motion of symbolic point along the loop (see Fig. 6). It can be found from the condition of equality of time τ_1 of motion of symbolic point along the arc of circumference and time τ_2 of progressive motion of the point symmetrically located on the circumference.

We have:

$$\tau_1 = 2\varphi_0, \qquad \tau_2 = \frac{R \sin \varphi_0}{a} T. \tag{2.12}$$

From the condition $\tau_1 = \tau_2$ we obtain the relationship for finding the radius of circumference $\bar{R} = Ra$ with assigned values of ϕ_0 and T:

$$\overline{R} = \frac{2\varphi_0}{T}\frac{1}{\sin\varphi_0}. \qquad (2.13)$$

3. Analysis of control of space vehicle by bank, optimum with respect to quick action.

Let us turn to the analysis of the optimum control of space vehicle by bank. We will examine motion of the vehicle relative to the center of mass on phase planes $\overline{\beta\Omega_\psi}$ and $\overline{\gamma\Omega_\gamma}$. From equations of motion (1.20) it follows that with the appropriate selection of controls the yaw motion of the space vehicle does not depend on its bank motion and can be analyzed separately.

Let us examine yaw motion of the vehicle:

$$\overline{\Omega}_\psi' = -\overline{\beta} + \overline{u}_\psi, \quad \overline{\beta}' = \overline{\Omega}_\psi. \qquad (3.1)$$

With $\overline{u}_\psi = 0$ motion of the vehicle on phase plane $\overline{\beta\Omega_\psi}$ is representei in the form of a circumference with center at point O. The radius of circumference depends on the initial conditions $\psi(0)$ and $\overline{\Omega}_\psi(0)$: $R = \sqrt{\overline{\beta}(0)^2 + \overline{\Omega}_\psi^2(0)}$. The symbolic point is displaced along the

circumferance in the clockwise direction of motion, completing a full revolution during $T_n = 2\pi$. Motion of the space vehicle with Rayleigh control $\bar{u}_\psi = \pm 1$ is represented on the phase plane of the circumferance, shifted along axis $0\bar{\beta}$ by +1 and -1 respectively (Fig. 7, a).

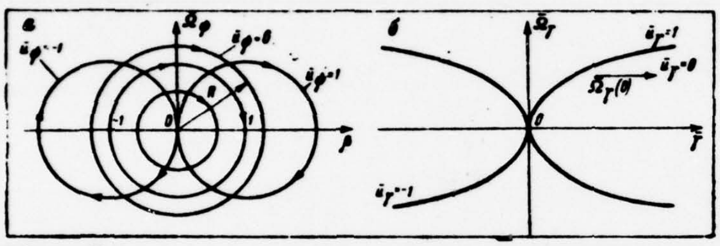


Fig. 7.

Bank motion of the vehicle (γ) generally, when $\sigma_\gamma^{\bullet} \neq 0$, depends on its yaw motion and on control $\bar{u}_\gamma$:

$$\bar{\Omega}_\gamma' = \sigma_\gamma^{\bullet}\bar{\beta} + \bar{u}_\gamma, \quad \bar{\gamma}' = \bar{\Omega}_\gamma. \tag{3.2}$$

In the case when $\bar{\beta} \equiv 0$, bank motion of the space vehicle on phase plane is described by parabola with $\bar{u}_\gamma \neq 0$ or by straight line $\bar{\Omega}_\gamma = \text{const}$ with $\bar{u}_\gamma = 0$ (Fig. 7, b).

Let us examine the problem about control of the space vehicle, optimum with respect to quick action, at zero initial conditions with respect to entire variable

$$\bar{\beta}(0)=\bar{\Omega}_{\psi}(0)=\bar{\Omega}_{\gamma}(0)=0,\ \bar{\gamma}(0)=0, \qquad (3.3)$$

providing turn of the vehicle to angle $\bar{\gamma}_0$ in minimum time T and reduction of it at the end of the turn to zero conditions

$$\bar{\beta}(T)=\bar{\Omega}_{\psi}(T)=\bar{\Omega}_{\gamma}(T)=0,\ \bar{\gamma}(T)=\bar{\gamma}_0. \qquad (3.4)$$

Such a system of boundary conditions must be satisfied, by analyzing the joint solution of equations of motion (3.1) and (3.2) with Rayleigh control $\bar{u}_{\psi}$ and $\bar{u}_{\gamma}$, determined taking into account the solutions of the system of conjugate equations (2.4) and (2.5).

From conditions of optimality it follows that control $\bar{U}_{\psi}$ - is linear, determined by change of the signs of conjugate functions $P_1(\tau)$. From Fig. 5 it follows that function $p_1(\tau)$ can have either two sections with different signs (Fig. 5, c) (two-pulse control), or four sections with different signs (four-pulse control) (Fig. 5, a, b). Three sections with different signs are also possible.

Let us examine symmetric control of the space vehicle with respect to bank and yaw. From analysis of the actual motion by the vehicle along angle β it follows that for satisfaction of boundary conditions (3.3) and (3.4), when T is not a multiple of 2π, the application of more than two pulses is necessary.

Fig. 8 shows an example of motion of the space vehicle on phase plane $\beta\Omega_{\gamma}$, satisfying boundary conditions of the problem. Let us show that the corresponding construction of the solution for conjugate variables $p_1(\tau)$ and $p_3(\tau)$ in this case is possible, and consequently, control is actually optimum.

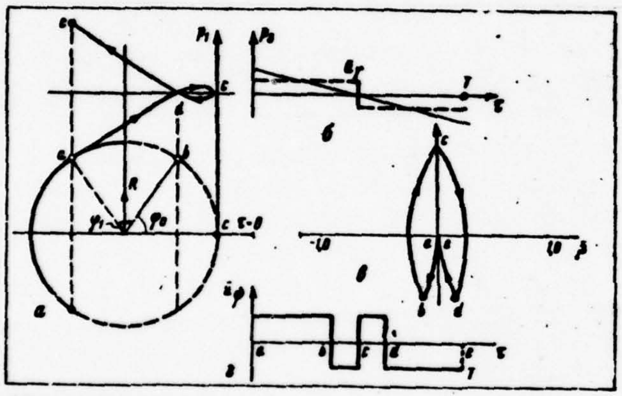


Fig. 8.

The solution for conjugate variable $p_3(\tau)$ is a linear function of time. In this case it is possible to accept that $p_3(0) = 1/\sigma_{\gamma}^{*}$, then

$$p_3(T) = -\frac{1}{\sigma_{\gamma}^{*}}, \qquad p_3(\tau) = \frac{1}{\sigma_{\gamma}^{*}} - \frac{2\tau}{T\sigma_{\gamma}^{*}}. \qquad (3.5)$$

Function $p_3(\tau)$ changes sign when $\tau = T/2$, and at this moment of time control $\bar{u}_{\gamma}$ changes sign (Fig. 8, b).

By assigning function $p_3(\tau)$ there is determined the position of the center of the circumference $p_1(p_2)$, in particular p_1 = -1 with τ = 0 and p_1 = +1 with τ = T.

From analysis of actual motion of the space vehicle there is known also the time of action of the second and third pulses (Fig. 8, d), which conform to change of variable $p_1(\tau)$ on loop bcd (Fig. 8, a). With the aid of relationship (2.13) by known time $\tau_1/2=\varphi_0$ is found the radius of circumference $\bar{R}$ for solution of conjugate equations. Having added angle ϕ_1, proportional to the duration of action of the first and fourth pulses, to angle ϕ_0, proportional to the time of action of the second and third pulse, we obtain the initial position of symbolic point on circumference $p_1(p_2)$.

Using the results, obtained in section 2, it is easy to show that change of $p_1(\tau)$ matches the required change of control U_ϕ and, consequently, optimum control, satisfying boundary conditions (3.3) and (3.4), is found.

As angle of turn $\bar{\gamma}_0$ increases, the time of the transient process T increases and accordingly the durations of the second and third pulses increase. With a certain time of transient process T the durations of the second and third pulses are maximum and with further growth of T start to diminish (Figs. 9 and 10).

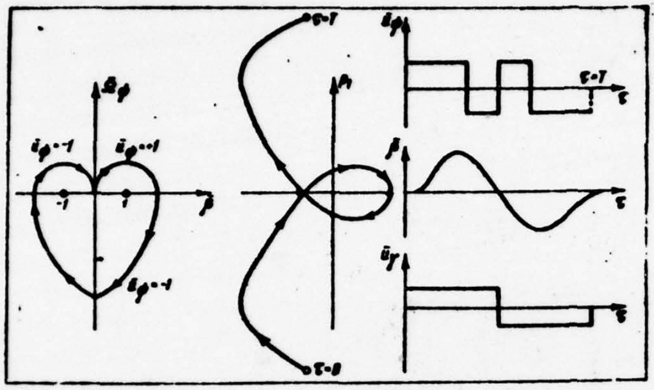


Fig. 9.

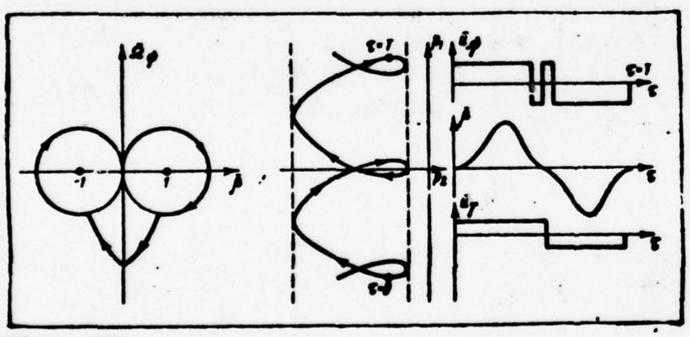


Fig. 10.

With $\bar{\gamma}_0$, corresponding to T = n4π (where n - whole number), control is boundary and is accomplished by two pulses (Fig. 11).

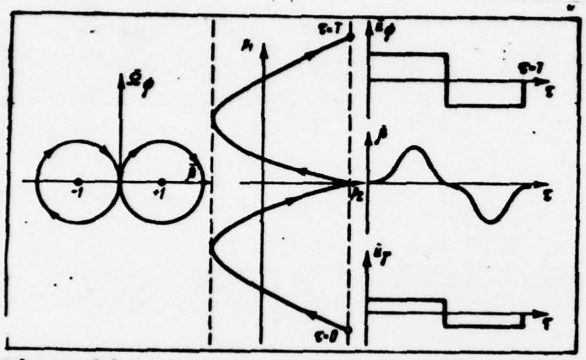


Fig. 11.

With further increase of $\bar{\gamma}_0$ (growth of T) the control is again accomplished by four pulses (Fig. 12).

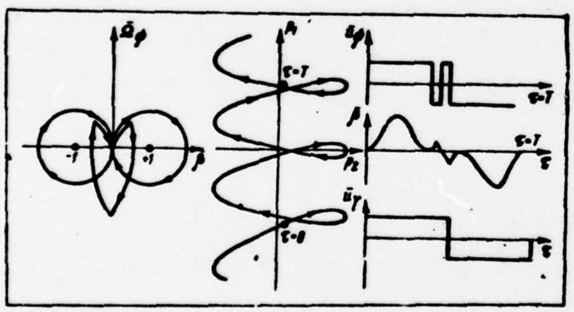


Fig. 12.

Analysis of three-pulse control (Fig. 13), satisfying boundary conditions (3.3) and (3.4), shows that although it is realizable, it is not optimum with respect to quick action of control.

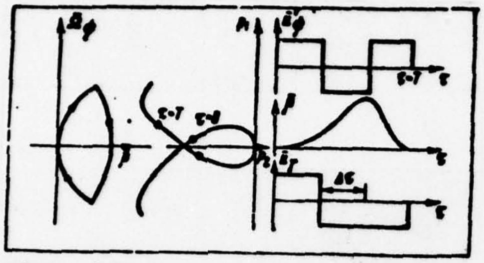


Fig. 13.

This is connected with the fact that besides the development of additional increase of $\bar{\gamma}_0$ due to $\bar{\beta}$, angular bank velocity is developed, the compensation of which leads to loss of the obtained gain and makes motion not optimum. In all the cases examined above (Figs. 8-12) the motion of the space vehicle along angle $\bar{\beta}$ is symmetric, i.e.,

$$\int_0^{T/2} \bar{\beta} d\tau = - \int_{T/2}^{T} \bar{\beta} d\tau. \tag{3.6}$$

As a result the additional angular bank velocity on time interval [0, T/2], caused by slip, is compensated on time interval

(T/2, T) by the further development of $\overline{\beta}$. Thanks to this the time intervals of action of positive and negative controls U_γ are identical.

For finding the connection between the value of $\overline{\gamma}_0$ and the minimum time of the transient process T let us convert equations (3.2). Let us represent $\overline{\Omega}_\gamma$ and $\overline{\gamma}$ in the form of the sum of two terms:

$$\overline{\Omega}_\gamma = \overline{\Omega}_{\gamma_1} + \overline{\Omega}_{\gamma_2}, \gamma = \gamma_1 + \gamma_2, \tag{3.7}$$

and variables $\overline{\gamma}_1, \overline{\gamma}_2, \overline{\Omega}_{\gamma_1}$ and $\overline{\Omega}_{\gamma_2}$ we will find from equations

$$\overline{\Omega}'_{\gamma_1} = \dot{\sigma}_\gamma \overline{\beta}, \ \overline{\gamma}'_1 = \overline{\Omega}_{\gamma_1}; \tag{3.8}$$

$$\overline{\Omega}'_{\gamma_2} = \bar{u}_\gamma, \ \overline{\gamma}'_2 = \overline{\Omega}_{\gamma_2}. \tag{3.9}$$

From equations (3.8) it follows that

$$\overline{\Omega}_{\gamma_1}(T) = \dot{\sigma}_{\gamma_0} \int_0^T \overline{\beta} d\tau, \ \overline{\gamma}_1(T) = \dot{\sigma}_\gamma \int_0^T \int \overline{\beta} d\tau \, d\tau. \tag{3.10}$$

Above, from analysis of $\ddot{\beta}(\tau)$, it was shown that $\int_0^T \overline{\beta} d\tau = 0$. Due to this equation (3.9) can be solved separately, not considering the equations (3.8). As a result the solution for $\overline{\gamma}_0(T)$ is written out in the form

$$\overline{\gamma}_0(T) = 2\left(\frac{T}{2}\right)^2 + \dot{\sigma}_\gamma \int_0^T \int \overline{\beta}(\tau, T) \, d\tau \, d\tau \, \frac{T^2}{2} + \gamma_2. \tag{3.11}$$

Relation (3.11) permits finding the dependence of $\overline{\gamma}_0$ on the time of the transient process T.

Function $\beta(\tau, T)$, entering the relation, depends on time T, since control a_+ depends on this value.

From relation (3.11) is easily determined the boundary value of bank angle $\bar{\gamma}_0$, corresponding to two-pulse control (Fig. 11):

$$\bar{\gamma}_0 = 4\pi^2 (2 + \dot{\alpha}_\gamma). \tag{3.12}$$

Generally, if time T is a multiple of 4π, the expression for $\bar{\gamma}_n$ is written out in the form

$$\bar{\gamma}_n = (2n)^2 \pi^2 (2 + \dot{\alpha}_\gamma), \tag{3.13}$$

where n - number of periods of natural oscillations with respect to $\bar{\beta}$ on half the time interval of the transient process.

Despite the simple equations of motion, finding the dependence $\bar{\gamma}_0(T)$ is a rather cumbersome operation. Let us write out some necessary relationships. From analysis of the geometric picture of motion on phase plane (Fig. 14) it is possible to obtain the following expressions for the main parameters:

$$R = \sqrt{5 - 4\cos\varphi_1}, \tag{3.14}$$

$$\varphi_2 = \operatorname{arctg} \frac{\frac{\sin\varphi_1}{2 - \cos\varphi_1} + 2\sqrt{1 - \cos\varphi_1}}{1 - \frac{2\sin\varphi_1}{2 - \cos\varphi_1}\sqrt{1 - \cos\varphi_1}}, \tag{3.15}$$

where ϕ_1 - angle, proportional to the time of action of the first

(fourth) pulse; ϕ_2 - angle, proportional to the time of action of the second (third) pulse.

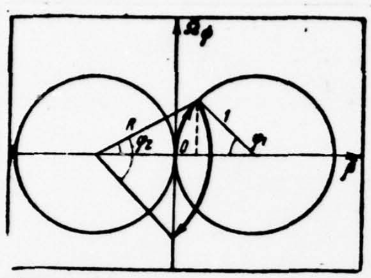


Fig. 14.

The time of transient process T is determined by formula

$$T = 2(\varphi_1 + \varphi_2). \qquad (3.16)$$

On Fig. 15 is constructed the dependence $\phi_2(\phi_1)$, and on Fig. 16 - dependence of $\bar{\gamma}_2/2\sigma_\gamma^*$ on ϕ_1, with the aid of which for each value of σ_γ^* it is possible to find the value of $\bar{\gamma}_0(T)$.

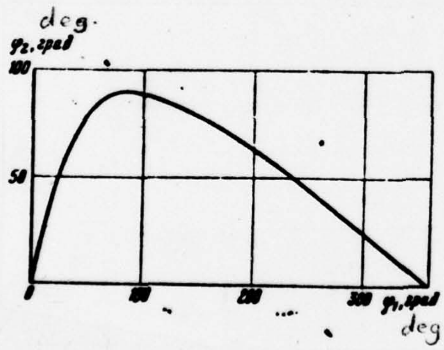


Fig. 15.

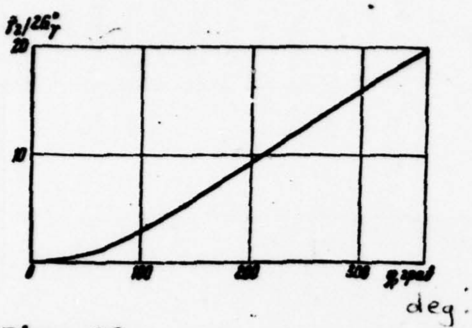


Fig. 16.

Let us estimate the value of "gain" in time ($\Delta\tau$), caused by simultaneous control of bank and yaw, in comparison with optimum control only by bank. Time ($T_0 + \Delta\tau$), required for turning the space vehicle to angle $\bar{\gamma}_1 + \bar{\gamma}_2$ with optimum isolated bank control, is estimated by formula

$$\gamma_1 + \gamma_2 = 2\left(\frac{T_0 + \Delta\tau}{2}\right)^2. \qquad (3.17)$$

where γ_1 - bank angle, caused by bank control with time-optimum maneuver, executed in time T_0; γ_2 - bank angle, providing slip with time-optimum maneuver; T_0 -time of execution of maneuver with time-optimum bank and yaw control.

From (3.17) we obtain the nonlinear dependence of time "gain" on the value of T_0 and γ_2:

$$\Delta\tau = -T_0 + \sqrt{T_0^2 + 2\gamma_2}. \qquad (3.18)$$

4. Optimum quick-action compensation of initial deviations of the space vehicle with respect to yaw and angle of attack.

Let us examine the problem of optimum quick-action compensation of deviations of the space vehicle with respect to yaw. As earlier, we will analyze the equations of motion

$$\bar{\Omega}_{\psi}' = -\bar{\beta}' + \bar{u}_{\psi}, \bar{\beta}' = \bar{\Omega}_{\psi}, \tag{4.1}$$

$$\bar{\Omega}_{\gamma}' = a_{\gamma}\bar{\beta} + \bar{u}_{\gamma}, \bar{\gamma}' = \bar{\Omega}_{\gamma} \tag{4.2}$$

with boundary conditions

$$\begin{gathered}\bar{\Omega}_{\psi}(0) = 0, \bar{\beta}(0) = \bar{\beta}_{0}, \bar{\Omega}_{\gamma}(0) = \bar{\gamma}(0) = 0, \\ \bar{\Omega}_{\psi}(T) = \bar{\beta}(T) = \bar{\Omega}_{\gamma}(\tau) = \bar{\gamma}(T) = 0.\end{gathered} \tag{4.3}$$

From equations of motion (4.1) and (4.2) we see that motion along angle $\bar{\beta}$ can be investigated independently from motion along angle $\bar{\gamma}$. Thus, the problem of compensation of initial deviation by angle $\bar{\beta}$ is broken down into the problem of optimum control of the vehicle with respect to yaw and problem of compensation of the accumulated error with respect to bank.

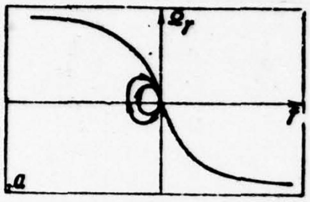
The solution of the first problem is known, it is examined in a number of works, moreover, its solution leads to synthesis of the

system, realizing algorithm of optimum control [7].

The complete problem about optimum control with respect to $\bar{\beta}$ and $\bar{\gamma}$, evidently, was not examined earlier.

It is necessary also to note that the problem about optimum quick-action compensation of deviation with respect to α coincides with the problem about compensation of deviation with respect to β [see equations (1.1) and (1.2)] and its solution, as was noted above, is known.

It is easy to show that the problem of optimum compensation of $\bar{\beta}$ and $\bar{\gamma}$ deviation may not have a single solution. This is connected with the fact that optimum control with respect to $\bar{\beta}$ does not depend on motion of the space vehicle along angle $\bar{\gamma}$ and has its "characteristic" time of transient process. At the same time, $\bar{\gamma}$ control depends on the motion of the vehicle with respect to yaw. When control by bank is ineffective and the transient process with respect to $\bar{\gamma}$ is slower than $\bar{\beta}$, control is unambiguous. In the case when effectiveness U_{γ} is great, angle $\bar{\gamma}$ deviation can be compensated by different methods, if the process was terminated in time assigned by motion $\bar{\beta}$. Examples of the noted motions are illustrated in Fig. 17.



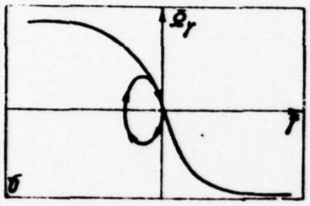


Fig. 17.

Let us examine in somewhat more detail the case when the solution of the formulated problem is unambiguous, namely: when control by $\bar{\gamma}$ is ineffective and the process of compensation of disturbances by $\bar{\gamma}$ from the motion of the space vehicle on yaw occupies more time than the process of compensation of motion by $\bar{\beta}$.

Analogously by relationship (3.7) we represent the change of $\bar{\gamma}$ consisting of two components: "forced" ($\bar{\gamma}_1$), caused by $\bar{\beta}$ disturbance, and "compensating" ($\bar{\gamma}_2$), caused by bank control:

$$\bar{\gamma}_1' = \bar{\Omega}_{\gamma_1}, \qquad \bar{\Omega}_{\gamma_1}' = \sigma_\gamma \bar{\beta}; \tag{4.4}$$

$$\bar{\gamma}_2' = \bar{\Omega}_{\gamma_2}, \qquad \bar{\Omega}_{\gamma_2}' = \bar{u}_\gamma. \tag{4.5}$$

The solution for $\bar{\gamma}$ is represented in the form

$$\bar{\gamma} = \bar{\gamma}_1 + \bar{\gamma}_2. \tag{4.6}$$

Since in the examined case the transient process with respect to β is finished earlier than the process with respect to $\bar{\gamma}(T_\beta < T_\gamma)$, then the solution of equations (4.4) can be taken as initial conditions for equations (4.5) and they can be written in the form

$$\bar{\gamma}_2(0) = \dot{\sigma}_\gamma \left(\int_0^{T_\beta} \int \bar{\beta} d\tau d\tau - T_\beta \int_0^{T_\beta} \bar{\beta} d\tau \right) = \dot{\sigma}_\gamma (I_2 - T_\beta I_1), \tag{4.7}$$

$$\bar{\gamma}_2'(0) = \dot{\sigma}_\gamma \int_0^{T_\beta} \bar{\beta} d\tau = \dot{\sigma}_\gamma I_1, \tag{4.8}$$

where $I_1 = \int_0^{T_\beta} \bar{\beta} d\tau$, $I_2 = \int_0^{T_\beta} \bar{\beta} d\tau d\tau$.

In expression (4.7) interval I_1 considers the change of $\bar{\gamma}$ on time interval $(0, T_\beta)$, caused by initial conditions on $\bar{\gamma}_2'(0)$ (4.8). Thus, the problem is reduced to known problem about control of isolated motion of a space vehicle by bank. The example of phase trajectory is illustrated by Fig. 18, a, on which is shown change of $\bar{\gamma}_2(\tau)$ and $\bar{\gamma}_1(\tau)$.

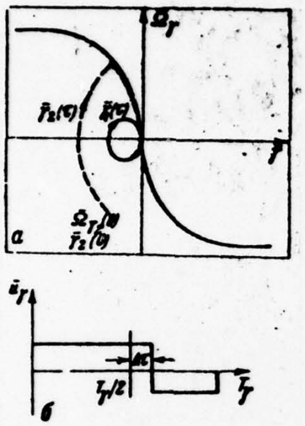


Fig. 18.

As is known, optimum quick-action control by bank is accomplished by two pulses (Fig. 18, b), the duration of which differs by value $\Delta\tau$, determined from the condition of compensation of initial angular velocity $\bar{\gamma}_2'(0)$:

$$2\Delta\tau = \bar{\gamma}_2'(0), \qquad \Delta\tau = \frac{I_1\dot{\alpha}_\gamma}{2}. \tag{4.9}$$

The total time of transient process with respect to bank T_γ is determined from the condition of compensation of total deviation with respect to bank, caused by development of $\bar{\beta}$:

$$\gamma_2^\Sigma = \dot{\alpha}_\gamma [I_2 + (T_\gamma - T_\beta) I_1]. \tag{4.10}$$

The value of T_γ is found from relationship

$$T_\gamma = 2\left[\Delta\tau + \sqrt{2(\Delta\tau)^2 + \bar{\gamma}_2}\right]. \tag{4.11}$$

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